5. - due Apr 27th 2006

- 1 An action, $J_a = 1/2\pi \oint p_x(x)dx$, is defined for a hamiltonian system with canonical co-ordinate x and momentum x.

  For the harmonic oscillator, $H = 1/2(\dot{x}^2 + \omega^2 x^2)$. Find the action.

- 2 Go to http://burro.astr.cwru.edu/JavaLab/index.html run the applets "GalCrash" and "SOS" using a range of different initial conditions, exploring the alternatives systematically.

  In a page or less, discuss your results. Show figures (on extra pages) as appropriate.

- 3 Assuming the solar neighbourhood has a density of $0.044\,\mathrm{M}_\odot\ \mathrm{pc}^{-3}$, a local one dimensional velocity dispersion of 30 $\mathrm{km\,s}^{-1}$, and that a typical field star has a mass of $1\mathrm{M}_\odot$ what is the maximum semi-major axis of a binary consisting of two $1\mathrm{M}_\odot$ stars which could survive for 10^{10} years?

  What would your answer be for two $0.5\mathrm{M}_\odot$ stars? What if the typical field stars has mass of $0.5\mathrm{M}_\odot$? (B&T 8-3)

  For a globular cluster with central density of 10^4 stars per cubic parsec, and a central one dimensional dispersion of 10 $\mathrm{km\,s}^{-1}$, at what semi–major axis is the hard–soft boundary for a binary consisting of a $1.4\mathrm{M}_\odot$ neutron star and a $0.7\mathrm{M}_\odot$ main sequence star, assuming the rest of the stars have a mass of $0.5\mathrm{M}_\odot$?

  What is your answer for the same system in a galactic nucleus of the same density but with a dispersion of 300 $\mathrm{km\,s}^{-1}$?

- 4 For a system of particles interacting through inverse square forces, find the relationship between the relaxation time and the crossing time assuming space has D dimensions, where D is a positive integer. (B&T 8-6)

  Near a black hole the density of stars rises as $r^{-3/2-p}$ while the dispersion rises as $r^{-1/2}$. Find how the relaxation timescale scales with radius.

  For isothermal distribution functions, theory predicts $p = 0$, comment.

- 5 Show that if two stars are drawn from a Maxwellian distribution, the probability distribution of their relative speeds is also a Maxwellian distribution. What is the dispersion of the relative velocity distribution? (This is B& T 7-3).

- 6 Estimate the relaxation time at the sun's position in the Milky Way. Estimate the relaxation time in the inner 1-10 pc of the centre of the Milky Way. Do you think the presence of a massive black hole in the centre will affect your estimate? If so, how?

- **Bonus Question:**

  Explain, briefly, what the underlying relation is between the Kepler potential and the 3-D Simple Harmonic Oscillator, which directly requires that they have the same number of independent isolating integrals. *Hint: the answer requires a mathematical statements, not just a qualitative description*