Calibrating RR Lyr and Cepheid Variables

In order to use RR Lyrae and Cepheids, the absolute luminosities of these objects must be calibrated. In the case of RR Lyr stars, this means determining the star's absolute magnitude (and perhaps how the absolute magnitude changes with metallicity). For Cepheids, one must determine both zero point and slope of the period-luminosity relation.

Numerous methods are used to estimate the absolute magnitudes of pulsating stars, including statistical parallax (whereby the Sun's motion through space provides the baseline), main-sequence fitting of clusters which contain the stars, and pulsation analysis theory. Here, we'll present one especially interesting technique: the Baade-Wesselink Method.

The Baade-Wesselink Method

Baade-Wesselink works for any pulsating star (or even exploding stars, such as supernovae). Consider a pusating star at minimum, with measured temperature, T_1, and observed flux, f_1. If the star's radius at minimum is R_1, then

$$f_1 = \frac{4\pi R_1^2 \sigma T_1^4}{4\pi D^2} \tag{6.01}$$

where D is the star's distance. Later on at maximum, the star has observed flux, f_2, a temperature T_2, and radius R_2, so

$$f_2 = \frac{4\pi R_2^2 \sigma T_2^4}{4\pi D^2} \tag{6.02}$$

The temperatures and fluxes are both observable, so there are 3 unknowns (R_1, R_2, and D) and two equations.

Now suppose you observe a star spectroscopically during its pulse from R_1 to R_2. During this time, the star's atmosphere has expanded at a velocity $v(t)$ from R_1 at time t_1 to R_2 at time t_2. So

$$R_2 = R_1 + \Delta R = R_1 + \int_{t_1}^{t_2} v(t)\, dt \tag{6.03}$$

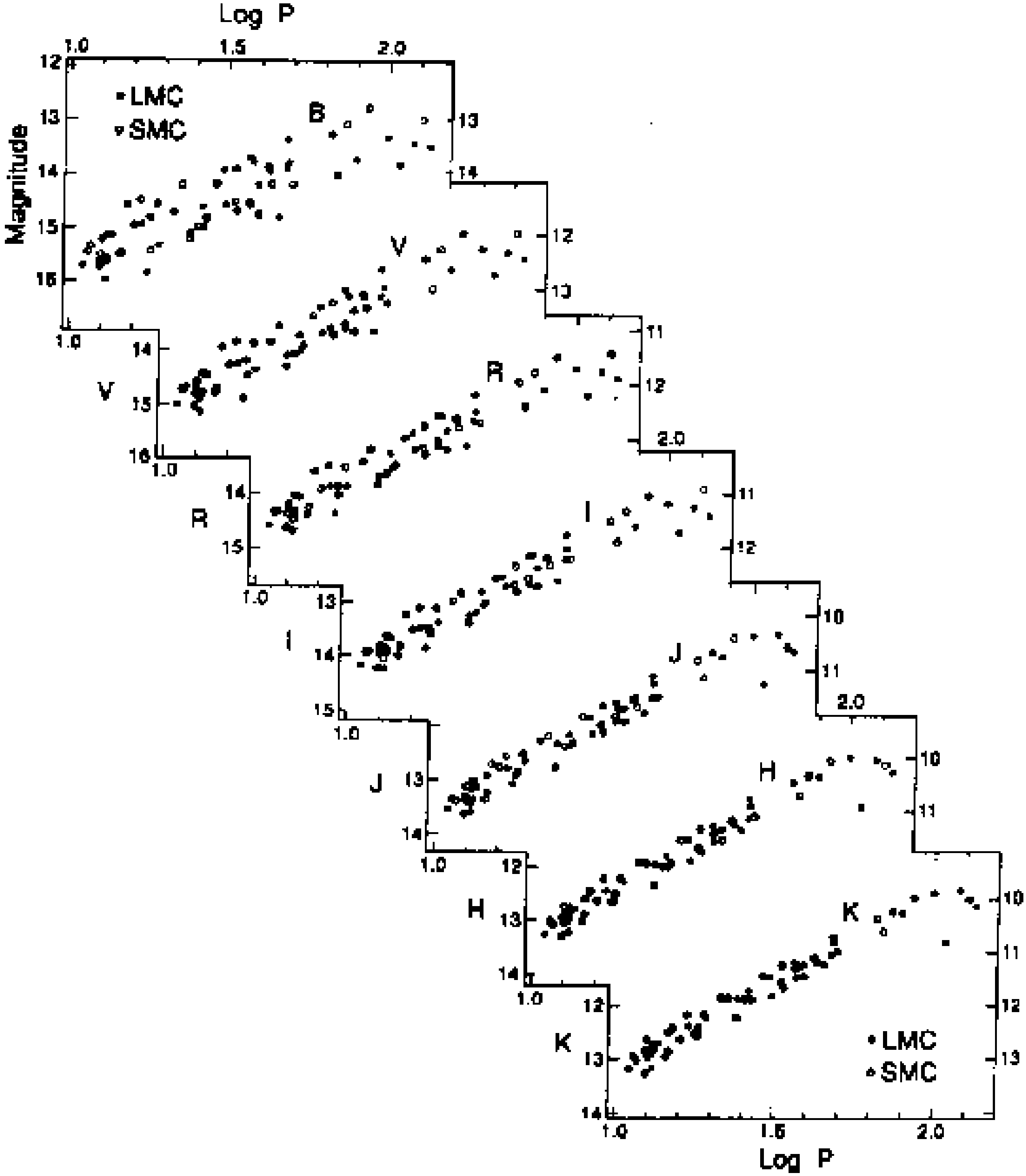
If you measure $v(t)$ throughout the pulse, you have a measure of ΔR. This gives you three equations and three unknowns, and let's you solve for D. (This is left as an exercise to the student with nothing better to do.)

In practice, the method is a bit more complicated, since a) the region of the star which forms the absorption line may be above or below the stellar photosphere, and b) only the center of the star has a radial velocity v; the radial velocity of the limb is zero. Consequently, much of the work associated with Baade-Wesselink is figuring out how these effects folds into the computation of $v(t)$.

The Slope of the Period-Luminosity Relation

With work, the Baade-Wesselink method and other techniques can be used to obtain distances to individual stars. But in the Milky Way, there are only a handful (~ 10) Cepheids whose distances (and therefore absolute magnitudes) with easily obtainable distances. (Remember, Cepheids are Pop I objects, so they are usually located in star-forming regions in the plane of the galaxy, surrounded by patchy dust.) Thus, Galactic Cepheids are not ideal for determining the slope of the Period-Luminosity Relation.

The best way to fix the slope is to observe a large sample of Cepheids at the same distance. Traditionally, this has been done in the Large Magellanic Cloud, our nearest (non dwarf) galaxy.



Log P
Magnitude
LMC
SMC
B
V
R
I
J
H
K
Log P

Note the increased scatter in the P-L relation at bluer wavelengths. Part of this is due to the larger effect of extinction. However, part of the is intrinsic. The period of a Cepheid depends on both the star's luminosity and its temperature, but because the instability strip is narrow, the latter variable is usually neglected. However, consider the case of two stars on opposite sides of the strip. When viewed in the optical, the small difference in temperature will propagate into a rather large difference in flux within the bandpass (since you're near the peak of the blackbody curve). This introduces scatter into the period-luminosity relation. On the other hand, if the two stars are viewed in the infrared, the flux within the bandpass doesn't depend that much on temperature, hence the scatter is smaller.

By observing Cepheids at multiple wavelengths, it is possible to remove all the effects of reddening. Consider a sample of Cepheids, extinguished by an amount of dust parameterized by $E(B-V)$. The period-luminosity relation says that

$$\langle M_x \rangle = a \log P + b \tag{6.04}$$

where x is the wavelength of the observation. When you observe Cepheids in another galaxy, you will observe their periods and their apparent magnitudes, $\langle m_x \rangle$. You therefore derive the apparent distance modulus, μ_x, which is related to the true distance modulus, μ_0, by

$$\mu_x = \langle m_x \rangle - \langle M_x \rangle = \mu_0 + R_x E(B-V) \tag{6.05}$$

In other words, you have one equation, but two unknowns, μ_0 and $E(B-V)$. However, if you observe at more than one wavelength, you have more than one equation (but still the same two unknowns). So you can solve for *both* the reddening and the distance simultaneously.

Secondary Standard Candles

Standard candles can be divided into several types. First, there are *primary* distance candles, such as parallax, moving cluster, and Baade-Wesselink measurements. These require no assumptions about their calibration. Next, there are *secondary* standard candles, that require a calibration via primary methods. These secondary methods include main-sequence fitting, Cepheids and RR Lyrae variables; without parallax, *etc.*, these would be useless.

The techniques we will now talk about are, by and large, *tertiary* standard candles. Most of them are calibrated in nearby galaxies using Cepheids and/or RR Lyrae stars. Hence the uncertainties in the primary and secondary methods will propagate into these measurements. Moreover, the uncertainties in the *testing* of tertiary standard candles can also become an issue.

There are two possible ways to test tertiary standard candles. The first, an external test, is compare the distances derived by one technique with those derived from another. If the two agree, then both methods are correct (or both are wrong in the same way.) Alternatively, you can perform an internal test and derive distances to large numbers of galaxies at the same distance (say, a bunch of galaxies in a cluster). If no systematic error is seen (such as the blue galaxies being closer than the red galaxies), then you can feel somewhat confident that the method is producing good relative distances.

Luminosity Functions

It is very common in astronomy to study the number of objects present in a sample versus magnitude. Such a plot is called a luminosity function. There are several luminosity function standard candles, including planetary nebulae, globular clusters, and red giant stars. In general, these methods do not go out far enough to be used to measure the undisturbed Hubble Flow, but they are useful for measuring distances to early-type galaxies (which have no Cepheids) or checking for systematic errors on the distance ladder.

Luminosity functions are extremely useful in astronomy, not only for distance measurements, but for a whole host of other problems, from the the distribution of Milky Way field stars to the evolution of galaxies in the universe. Thus, before addressing the individual techniques, we should first consider them in a general sense.

There are two ways to think of a luminosity function. The conventional way is just to consider it as a function, $N(m)$, where N is the number of objects within a finite bin size. (Actually, one almost always uses $\log N$, rather than N.) Under this interpretation, one chooses the bin size, and creates the luminosity function by placing each object into its appropriate bin. Note, however, that due to observational errors, the observed luminosity function will *not* be the intrinsic function.

To see this, consider an intrinsic power-law luminosity function, where each bin has 10 times the counts in the previous bin, *i.e.,* bin k has 10 counts, bin $k+1$ has 100 counts, bin $k+2$ has 1000 counts, *etc.* In the presence of observational errors, some of the counts from each bin will spill over into the adjacent bins, so bin

$k+1$ will lose a fraction of its counts to bins k and $k+2$, but also gain counts from these bins. But note: all is not symmetrical. If bin $k+1$ loses 5% of its counts to bin $k+2$, but gains 5% of bin $k+2$'s counts, then bin $k+1$ is a big winner. We will therefore overestimate its numbers. The observed luminosity function is a *convolution* of the true function with the photometric error function. In general, the observed function will always be smoother and flatter than the true function.

The Eddington Correction

There's a very tricky way to handle observational errors, which was first used by Eddington in 1913, and which is sometimes called the Eddington correction. (By far the best place to read about it is the classic 1953 book *Statistical Astronomy,* by Trumpler and Weaver.) Let $N_{obs}(m_0)$ be the observed number of counts in a bin centered at m_0, and $N_t(m)$ be the true luminosity function. To save on notation, let $x = m - m_0$, and $G(x)$ be the function which describes how the true number of counts in the bin centered at m_0 is distributed. For simplicity, let's also assume that $G(x)$ is symmetrical about zero, and, of course, normalizes to one, since no objects are lost. (Usually, you can consider G to be a Gaussian, with some dispersion, σ). With this notation, N_{obs} is just the convolution of N_t with $G(x)$,

$$N_{obs} = N_t \circ G \tag{6.06}$$

and

$$N_{obs}(m_0) = \int_{-\infty}^{\infty} N_t(m_0 - x)\, G(x)\, dx \tag{6.07}$$

Now, the key to solving this equation is to expand N_t about m_0 using a Taylor series

$$N_{obs}(m_0) = \int_{-\infty}^{\infty} \left\{ N_t(m_0) - \frac{N_t'(m_0)}{1!} x + \frac{N_t''(m_0)}{2!} x^2 + \ldots \right\} G(x) dx \tag{6.08}$$

or

$$N_{obs}(m_0) = N_t(m_0) \int_{-\infty}^{\infty} G(x)\, dx - \frac{N_t'(m_0)}{1!} \int_{-\infty}^{\infty} x\, G(x)\, dx + \frac{N_t''(m_0)}{2!} \int_{-\infty}^{\infty} x^2 G(x)\, dx + \ldots \tag{6.09}$$

Note that integrals of the form

$$\mu_n = \int_{-\infty}^{\infty} x^n G(x) dx \tag{6.10}$$

give the *moments* of G. (The concept of moments of functions pops up again and again in astronomy.) So (6.10) can be re-written as

$$N_{obs}(m_0) = N_t(m_0) - \frac{\mu_1}{1!} N_t'(m_0) + \frac{\mu_2}{2!} N_t''(m_0) - \ldots \tag{6.11}$$

This is a perfectly good equation for $N_{obs}(m_0)$, except that you don't know $N_t(m_0)$, or its derivatives. But it is possible to be tricky, and say that for some set of variables, A, the following must be true

$$N_t(m_0) = N_{obs}(m_0) + A_1 N_{obs}'(m_0) + A_2 N_{obs}''(m_0) + \ldots \tag{6.12}$$

Now substitute for $N_t(m_0)$ in (6.11) using (6.12), and use (6.11) to find expressions for $N_{obs}'(m_0)$, $N_{obs}''(m_0)$, *etc.* In order for $N_{obs}(m_0)$ to equal $N_{obs}(m_0)$, each other term must equal zero. So,

$$\begin{aligned} A_1 - \frac{\mu_1}{1!} &= 0 \\ A_2 + \frac{\mu_2}{2!} &= 0 \\ A_3 - \frac{\mu_3}{3!} &= 0 \\ A_4 + A_2 \frac{\mu_2}{2!} + \frac{\mu_4}{4!} &= 0 \end{aligned} \tag{6.13}$$

This gives us the values of A. Finally, there is one last simplification. If $G(x)$ is symmetrical, all the odd numbered terms go to zero. (You can see this easily if you graph what's going on). So

$$N_t(m_0) = N_{obs}(m_0) - \frac{\mu_2}{2!} N''_{obs}(m_0) + \left\{ \left(\frac{\mu_2}{2!} \right)^2 - \frac{\mu_4}{4!} \right\} N^{iv}_{obs}(m_0) + \dots \tag{6.14}$$

And note: if $G(x)$ is a Gaussian function, $\mu_n = \sigma^n$. So, if we just keep the first term

$$N_t(m_0) = N_{obs}(m_0) - \frac{\sigma^2}{2!} N''_{obs}(m_0) \tag{6.15}$$

Luminosity Functions as Probability Distributions

A second way of looking at a luminosity function is to consider it as a probability distribution between the observed limits m_1 to m_2. For every observed galaxy, there is a 100% probability that its magnitude lies between m_1 and m_2, so

$$\int_{m_1}^{m_2} P(m)\, dm = 1.0 \tag{6.16}$$

and, with this normalization, the probability of observing an object with a magnitude between any two magnitudes m_a and m_b is

$$P(m_a < m < m_b) = \frac{\int_{m_a}^{m_b} P(m)\, dm}{\int_{m_1}^{m_2} P(m)\, dm} \tag{6.17}$$

In this case, the function $P(m)$ is the intrinsic luminosity function convolved with the photometric errors, *i.e.,* (6.06). The probability of finding any specific combination of objects is

$$P_t = \Pi\, P(m) \tag{6.18}$$

Under this formulation, the most-likely function of N_t is that which maximizes the total probability. You can therefore hypothesize different forms for N_t and choose the one that maximizes probability.

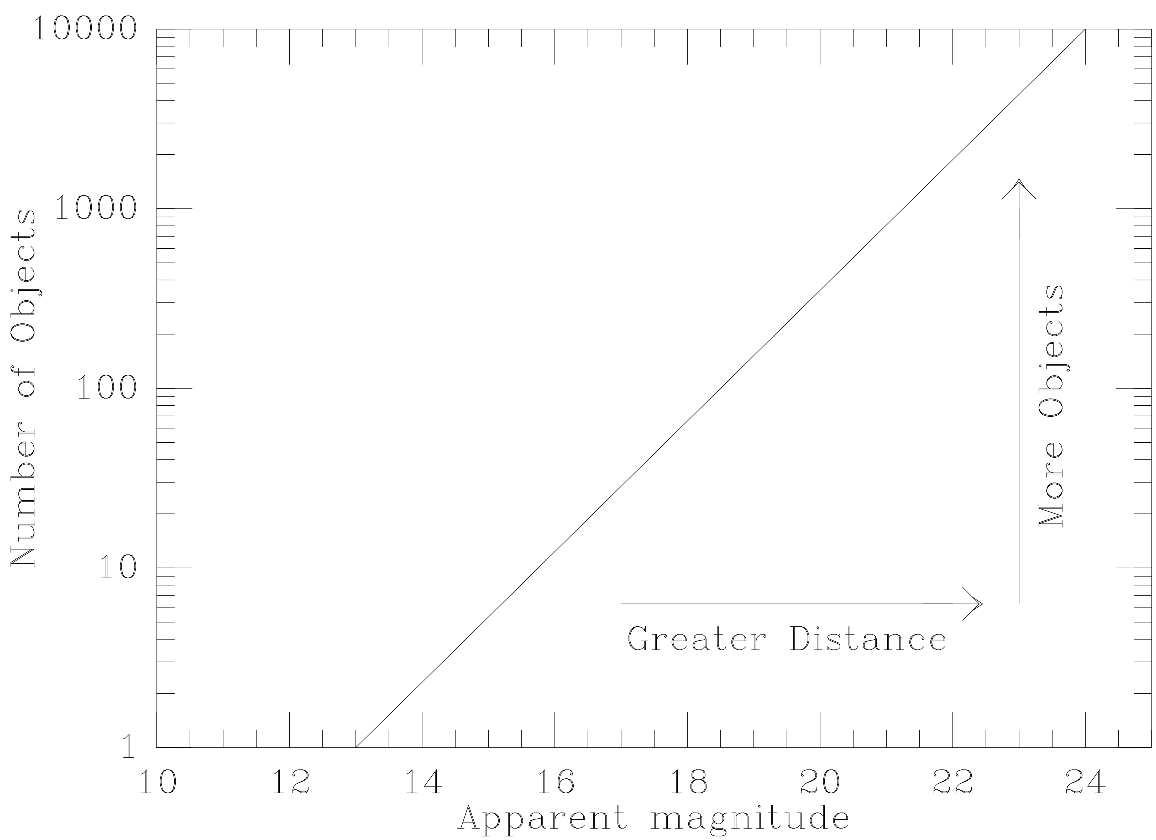
Some problems in astronomy are simple when you think of luminosity functions as probability functions, and impossible if you work with them as luminosity functions. (For instance, how do you bin a total of 5 objects?) And the reverse is also true. With practice, you can learn to recognize when to use which type of analysis.

Power-law Luminosity Functions

Consider a luminosity function which is a power law, *i.e.,* $N(\mathcal{L}) \propto \mathcal{L}^{\alpha}$, or

$$N(m) = am + b \tag{6.19}$$

where a and b are two constants. In general, of course, these constants are not known ahead of time. (In fact, they are generally what you are looking for.) Now, let's observe two galaxies that look identical on the sky, but let one be 10 times further away than the other. Obviously, the objects in the more distant galaxy will five magnitudes fainter than that of the nearer galaxy. But that more distant galaxy (in order to have the same apparent magnitude) would have to be 100 times more luminous. Therefore, the number of objects at each magnitude would be 100 times greater. Consequently, the $\log N$ *vs.* m diagram for both galaxies would be the same! This is a general rule – you can't tell anything from a power-law luminosity distribution.

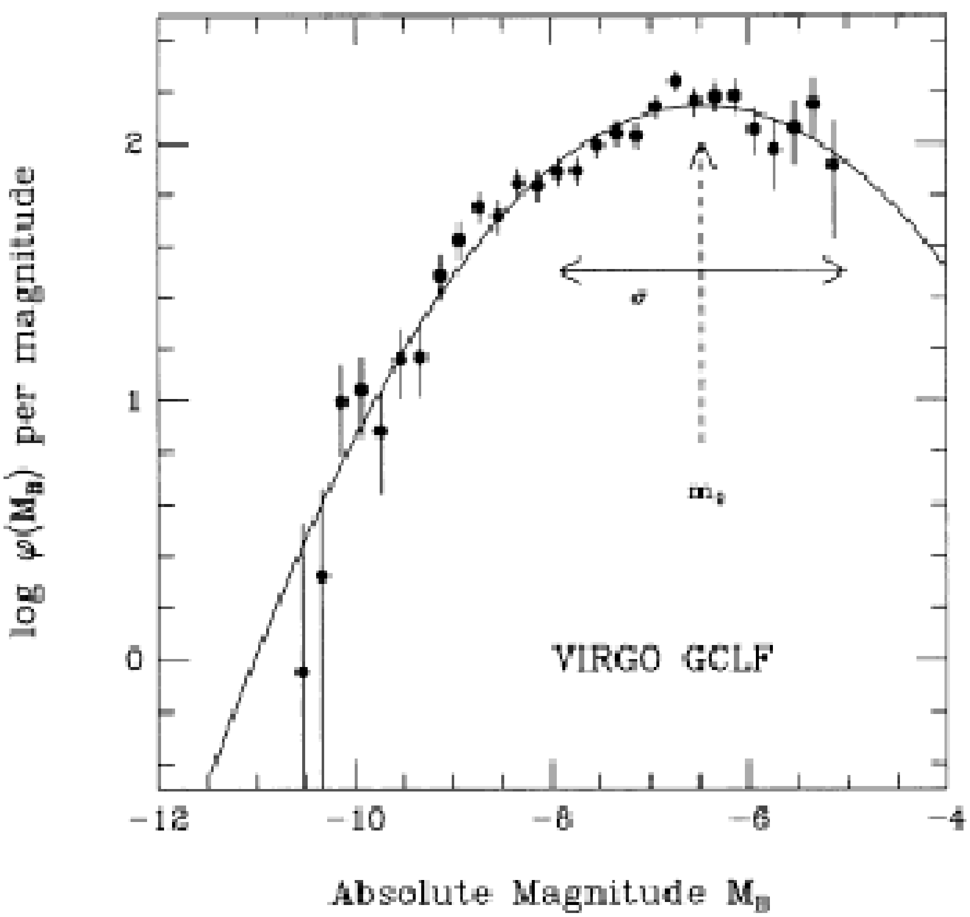


The Globular Cluster Luminosity Function

When put at the distance of $\gtrsim 1$ Mpc away, globular clusters look like a faint stars. But some galaxies have alot of them, making them easy to count, even with background contamination. (This is especially true in smooth, elliptical and S0 systems.) In most galaxies, the GCLF is a log Gaussian, *i.e.*, when plotted as $\log N$ *vs.* m, it is a Gaussian,

$$\log N(m) \propto e^{(M-M_0)^2/2\sigma^2} \tag{6.20}$$

with $M_0 \sim -7.3$ in the V-band, and $\sigma \approx 1.2$. Thus by observing the apparent magnitude of the GCLF peak, m_0, it's possible to estimate distance.



The observed globular cluster luminosity function for M87, and the best-fit Gaussian.

Tip of the Red Giant Branch

According to stellar evolution, when a star runs out of hydrogen in its core, it develops an inert helium core and evolves to the red in the HR diagram. Once on the giant branch, the stars increase their brightness, until helium itself fuses, via the triple-α process. Due to the physics of stellar interiors, this fusion takes place at about the same absolute luminosity for all stars with initial masses less than $\sim 2.5\mathcal{M}_\odot$. Thus, the tip of the red giant branch (TRGB) can be a useful standard candle.

Detecting individual red giants in distant galaxies is difficult; measuring their brightness is even harder. (Galaxies have many, many, many red giants, and their fluxes blur together.) In addition, although the total luminosity of the red giant stars do not depend on their mass or metallicity, the flux in a particular filter does. (For example, metal-poor TRGB stars are more luminous in the I band than metal-rich TRGB stars.) Nevertheless, by finding the apparent magnitude of the RGB tip, and comparing it to the absolute magnitude of the TRGB in galactic globular clusters, distances can be estimated.