Collisional Cooling and Detailed Balance

If every collisional excitation from the ground level to level j were followed (eventually) by the emission of a photon (or multiple photons) which leave the nebula, then the collisional cooling rate would be

$$L_C = N_e N_i \, q_{1j} \; \cdot \; h\nu_{1j} \tag{21.01}$$

But, in practice, collisional de-excitation also occurs. Consider a two-level atom of an ion. Let N_1 be the number of ions in the ground state, and N_2 be the number of ions in the excited state. In equilibrium, the number of electrons going out of N_1 is equal to the number of electrons entering level N_1, either by collisions or by radiative decay. In other words,

$$N_e N_i q_{12} = N_e N_2 q_{21} + N_2 A_{21} \tag{21.02}$$

The relative level population is therefore

$$\left(\frac{N_1}{N_2} \right) = \frac{N_e q_{12}}{N_e q_{21} + A_{21}} \tag{21.03}$$

The collisional cooling rate for this two-level ion is therefore

$$L_C = N_2 A_{21} \, h \, \nu_{21} = \frac{N_1 N_e q_{12}}{N_e q_{21} + A_{21}} \, A_{21} \, h \, \nu_{21} \tag{21.04}$$

Let's look at this equation a bit more closely. If we divide both the numerator and denominator of (21.04) by A_{21}, then

$$L_C = N_e N_1 q_{12} \, h \, \nu_{12} \left\{ \frac{1}{\frac{N_e q_{21}}{A_{21}} + 1} \right\} \tag{21.05}$$

Now note the limits. As the electron density, $N_e \rightarrow 0$, collisional de-excitation becomes unimportant, and the cooling rate becomes

$$L_C \longrightarrow N_e N_1 q_{12} \, h \, \nu_{12} \tag{21.06}$$

In this case, the cooling is proportional to the electron density, and every collision upward creates a photon which cools the nebula. On the other hand, as $N_e \rightarrow \infty$,

$$L_C \longrightarrow N_1 \left(\frac{q_{12}}{q_{21}}\right) A_{21} h\, \nu_{21} = N_1 \left(\frac{\omega_2}{\omega_1}\right) e^{-\Delta E/kT} A_{21} h\, \nu_{21} \tag{21.07}$$

This is nothing more than the cooling rate for a gas in thermodynamic equilibrium.

Of course, although a few atoms (such as Be II) can be treated with two levels, most require more. In fact, the p^2, p^3, and p^4 configurations all have 5 low-lying levels, separated with two major divisions. To calculate the cooling from an n-level atom, you have to solve a set of linear equations, equating the number of electrons entering a level to the number of electrons exiting the level.

$$\sum_{j \neq i} N_e N_j q_{ji} + \sum_{j > i} N_j A_{ji} = \sum_{j \neq i} N_e N_i q_{ij} + \sum_{j < i} N_i A_{ij} \tag{21.08}$$

The first sum is the number of electrons entering level i through collisions from all the other levels. The second term is the number of electrons entering level i through radiative decays from the levels above i. On the other side of the equal sign is the number of electrons exiting level i via collisions, and the number of electrons exiting level i via radiative decays downward. This is the equation of **detailed balance.**

Note that there is actually one other equation to detailed balance

$$\sum_{i}^{n} N_i = N \tag{21.09}$$

This simply says that the sum of the ions in all the levels must add up to the total number of ions there are. So this gives you $n+1$ equations, and n unknowns. Thus, with some algebra (and a little bit of patience) you can solve for the number of ions in each level.

Once the ion densities, are known, the collisional cooling follows simply from

$$L_C = \sum_i N_i \sum_{j<i} A_{ij}\, h\, \nu_{ij} \tag{21.10}$$

The equation of detailed balance also allows us to specify when collisions out of a level are more important than radiative transfers out of a level. From the left side of (21.08), this occurs when

$$\sum_{j \neq i} N_e N_j q_{ji} > \sum_{j<i} N_j A_{ji} \tag{21.11}$$

or, more to the point,

$$N_e(\mathrm{crit}) > \frac{\displaystyle\sum_{j<i} A_{ij}}{\displaystyle\sum_{j \neq i} q_{ij}} \tag{21.12}$$

$N_e(\mathrm{crit})$ is called the critical density for the level. If $N_e < N_e(\mathrm{crit})$ then collisional de-excitation is unimportant.

Thermal Equilibrium

A nebula in thermal equilibrium has

$$G = L_{ff} + L_R + L_C \tag{21.13}$$

where G is the heating due to photoionization (equations 20.06 and 20.08), L_{ff} is the cooling from free-free emission (20.09), L_R is the cooling from electron recombination (20.10), and L_C is the cooling due to collisional excitations (21.10). In the low density limit, each of these terms is proportional to electron density, so N_e cancels out. In this case, the nebular temperature will only depend on the energy of the ionizing photons and on the ionic abundances. At high densities, however, collisional de-excitations will decrease the efficiency of collisional cooling. Therefore, all things being equal, denser nebulae will be hotter.

One very important point to consider is that collisions are, by far, the most important mechanism for nebular cooling. Naturally, the amount of collisional cooling is proportional to the metal abundance, N_Z. But note: if N_Z is decreased, then the temperature of the nebula will rise considerably. However, recall that the equation for collisional excitation is

$$q_{ij} = 8.629 \times 10^{-6} \; \frac{\Omega(i,j)}{\omega_i T_e^{1/2}} e^{-\Delta E/kT} \text{ cm}^3 \text{ s}^{-1} \tag{20.15}$$

so if T rises, the exponential term $e^{-\Delta E/kT}$ will also rise, and therefore q_{ij} will rise. As a result, it is possible that decreasing the number of ions will actually increase the amount of ionic emission from the nebula.

Collisional cooling is always most efficient for ions where $\Delta E \approx kT$. If $\Delta E \gg kT$, then very few electrons will be excited, so little cooling will occur; if $\Delta E \ll kT$, then the amount of energy released in the downward transition will be inconsequential.