

## Photo-ionization of Heavy Elements

An insignificant amount of the central star's photons go into ionizing the heavy elements. The photo-ionization cross section of these elements is complicated, since 1) the experimental values for the cross sections are not well determined, 2) the cross sections are not smooth functions, due to resonances, 3) other effects can place the electrons of metals into excited states, from which the ionization can occur. Metal recombination rates are similarly uncertain, due to effects such as dielectronic recombination. (Imagine a recombination where the photon produced via the recombination goes directly into exciting one of the ground-state electrons. If this happens, the atom can have two excited electrons at once, with a total excitation energy greater than the ionization energy. If this occurs, one electron may give its energy to the other, causing the atom to re-ionize itself!)

Below are some of the energies (in eV) needed to ionize metals:

| Element  | II   | III  | IV   | V    |
|----------|------|------|------|------|
| Hydrogen | 13.6 |      |      |      |
| Helium   | 24.6 | 54.4 |      |      |
| Carbon   | 11.3 | 24.4 | 47.9 | 64.5 |
| Nitrogen | 14.5 | 29.6 | 47.4 | 77.7 |
| Oxygen   | 13.6 | 35.1 | 54.9 | 77.4 |
| Neon     | 21.6 | 41.0 | 63.5 | 97.1 |

Note that to strip one electron off oxygen requires 13.6 eV — exactly the same as hydrogen. Thus, when hydrogen is ionized, oxygen is ionized.

## Nebular Heating and Thermal Equilibrium

Consider the energy contained in the free electrons of a nebula. Every ionization creates a photo-electron with energy

$$\frac{1}{2}m_e v_p^2 = h(\nu - \nu_0) \quad (20.01)$$

while every recombination removes  $\frac{1}{2}m_e v_r^2$ . The difference between the mean energy of a photoionized electron, and that of a recombination electron must be lost by radiation if the nebula is to be in thermal equilibrium.

There is only one source of heating for a nebula: the photo-ionization of electrons. It is not difficult to estimate the initial temperature ( $T_i$ ) of these electrons. For a pure hydrogen nebula ionized by a blackbody the average energy of a photoionized electron is

$$\frac{3}{2}kT_i = \frac{\int_{\nu_0}^{\infty} \frac{J_\nu}{h\nu} h(\nu - \nu_0) d\nu}{\int_{\nu_0}^{\infty} \frac{J_\nu}{h\nu} d\nu} \quad (20.02)$$

One can find an analytic expression for this, if one assumes that the ionizing spectrum is a blackbody, and all the ionizing photons are on the Wien part of the blackbody curve, *i.e.*,  $kT_* < h\nu_0$ . In this case,

$$J_\nu = B_\nu(T_*) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \approx \frac{2h\nu^3}{c^2} e^{-h\nu/kT} \quad (20.03)$$

If you make this substitution, and crank through the math, then you find that

$$\frac{3}{2}kT_i \approx kT_* \implies T_i \approx \frac{2}{3}T_* \quad (20.04)$$

Two things to note. First,  $T_i$  depends only the shape of the ionizing radiation, not on its strength. (In other words, the thermal energy of an ionized electron depends only on the energy of the photon that hit it, not on how many photons there are.) Thus, the temperature of a nebula being ionized by 100 large 30,000 K stars will be less than that ionized by one 100,000 white dwarf. (The size of the nebula will be larger, but the temperature will be smaller).

Second, because the cross section to photoionization decreases as  $\nu^{-3}$ , high energy photons will penetrate deeper into a nebula than low energy photons. Thus,  $T_i$  may increase as the distance from the star increases.

The exact heating of a nebula is not difficult to compute. If  $G(H)$  is the energy gained by the nebula through the photoionization of hydrogen, then

$$G(H) = N(H^0) \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} h(\nu - \nu_0) a_\nu d\nu \quad (20.05)$$

which, through photoionization equilibrium, is

$$G(H) = N_e N_p \alpha_A \frac{\int_{\nu}^{\infty} \frac{J_\nu}{h\nu} h(\nu - \nu_0) a_\nu d\nu}{\int_{\nu}^{\infty} \frac{J_\nu}{h\nu} a_\nu d\nu} \quad (20.06)$$

Since the last term is simply injected energy,

$$G(H) = N_e N_p \alpha_A \left( \frac{3}{2} k T_i \right) \approx N_e N_p \alpha_A k T_* \quad (20.07)$$

A similar equation holds for helium, so the total heating is

$$G = G(H) + G(He^0) + G(He^1) \quad (20.08)$$

The amount of metals in the nebula is so little, that heating from those elements can easily be neglected.

## Sources of Cooling

There are three different mechanisms a nebula has for cooling itself. The first is through thermal bremsstrahlung of the free electrons; the full expression for this is

$$L_{ff} = \frac{2^5 \pi e^6 Z^2}{3^{3/2} h m_e c^3} \left( \frac{2\pi k}{m_e} \right)^{1/2} T^{1/2} g_{ff} N_e N_+$$

$$= 1.42 \times 10^{-27} Z^2 T^{1/2} g_{ff} N_e N_+ \text{ ergs cm}^{-3} \text{ s}^{-1} \quad (20.09)$$

where  $g_{ff}$  is the gaunt factor. Since virtually all of these photons are longward of  $\nu_0$  (most are in the radio), all this free-free luminosity leaves the nebula. However, this energy is mostly inconsequential. Free-free emission is the least important of all energy loss mechanisms.

A second source of cooling comes from the recombination of electrons into hydrogen (and helium). This is calculated simply from the kinetic-energy weighted recombination coefficients to each level of the atom

$$L_R(H) = N_e N_p kT \beta_A \quad (20.10)$$

where

$$\beta_A = \sum_{n=1}^{\infty} \sum_{L=0}^{n-1} \beta_{nL} \quad (20.11)$$

A typical value for  $\beta$  is  $10^{-13} \text{ cm}^3 \text{ s}^{-1}$ . Note that even though  $\beta$  represents the energy released by a recombined electron, it does not have the units of energy. By definition, the electron energy has been divided by  $kT$ , which is why (20.10) has a  $kT$  in the expression.

Recombination cooling is significant, but it does have a very interesting property. Recall from our discussion about recombination (15.12) that, due to electrostatic focusing, slow moving electrons are more likely to recombine than fast moving electrons. Thus, recombinational cooling will continually remove electrons from the low-energy tail of the Maxwellian distribution. Consequently, if this were the only source of nebular cooling, the temperature of the free electrons would actually increase!

This same effect makes the on-the-spot approximation for thermal balance less accurate than it is for ionization balance. (The slow moving electrons from the diffuse field are more important.) Still, one can use  $\beta_B$ , instead of  $\beta_A$  in (20.10) and get a reasonable approximation for recombinational cooling.

## Collisional Cooling

The most important physical process that occurs in an emission line region is collisional cooling. Consider an electron in some atom in state  $nL$ . If  $nL$  is not the ground state, that electron may decay, at a rate determined by the  $A$  value of the transition. If  $nL$  is the ground state, then, of course, the electron will just sit there. However, if the density of particles is high, or if the  $A$  value for downward transitions are low, a free electron may come by and collide the atom out of state  $nL$  into state  $n'L'$  before a spontaneous transition can occur. If state  $n'L'$  has less energy than state  $nL$ , then the free electron must carry away the energy. However, if state  $n'L'$  has  $\Delta E$  more energy than state  $nL$  (as it must, if the bound electron is already in its ground state), then the colliding electron will lose that amount of energy in the collision. The nebula will have therefore cooled.

The point to notice about this effect is where it occurs. The energy difference between the ground state of hydrogen and the first excited state is  $\Delta E = 10.2$  eV. Therefore, for collisional cooling to be important for hydrogen,  $kT \sim 10.2$  eV, or  $T \sim 120,000$  K. Since this is much hotter than the temperature of the free electrons, collisions with hydrogen atoms do not cool the nebula.

For helium, the energy difference between the ground state and the  $n = 2$  state is even higher (19.8 eV), so likewise, ground-state helium collisions are unimportant. Collisional cooling on metals, however, is another story. Common ions of oxygen, nitrogen, carbon, neon, and argon all have energy levels that are  $\sim kT$  above their ground state. Thus, they are extremely efficient for cooling the nebula.

To understand how efficient these ions are, we must consider the rate at which these collisions occur. The computation of the rates has four terms:

- 1) a term which includes the density of electrons ( $N_e$ ), and the density of ions ( $N_i$ ).
- 2) a term which includes the volume being swept out by a free electron each second ( $\sigma \propto v$ ) and the effects of electrostatic focusing ( $\sigma \propto v^{-2}$ ). In terms of temperature, this means that the rate of collisions must be

$$q \propto T_e^{-1/2} \quad (20.12)$$

- 3) a term which includes the Boltzmann factor. For a free electron with velocity  $v$ , if  $\Delta E > m_e v^2/2$ , then clearly, there is not enough energy in the collision to move the electron. Similarly, in terms of temperature, the larger the value of  $\Delta E$ , the fewer the number of electrons that can cause an excitation. On the other hand, if the collision involves a downward transition, there is no energy barrier. Thus

$$q \propto e^{-\Delta E/kT} \quad \text{if } \Delta E > 0 \quad (20.13)$$

- 4) a term which involves the statistical weights. If level  $n'L'$  is actually composed of 10 different states, then the collisional rate out of that level will be 10 times smaller than that of a similar level with only one state.
- 5) a (dimensionless) quantum mechanical cross section, which depends on the wave functions of the states.

If we put these terms together, the rate of collisions from state  $i$  to state  $j$  is

$$N_e N_i q_{ij}$$

where

$$\begin{aligned} q_{ij} &= \left( \frac{2\pi}{kT_e} \right)^{1/2} \frac{\hbar}{m_e^{3/2}} \frac{\Omega(i, j)}{\omega_i} \\ &= 8.629 \times 10^{-6} \frac{\Omega(i, j)}{\omega_i T_e^{1/2}} \text{ cm}^3 \text{ s}^{-1} \end{aligned} \quad (20.14)$$

if state  $j$  has lower energy than state  $i$  (*i.e.*,  $\Delta E < 0$ ). If, on the other hand,  $\Delta E > 0$ , then

$$q_{ij} = 8.629 \times 10^{-6} \frac{\Omega(i, j)}{\omega_i T_e^{1/2}} e^{-\Delta E/kT} \text{ cm}^3 \text{ s}^{-1} \quad (20.15)$$

Interestingly, the quantum mechanical collision strength,  $\Omega$ , is symmetrical, *i.e.*,  $\Omega(i, j) = \Omega(j, i)$ . This causes the two collisional rates to be related

$$q_{ij} = \frac{\omega_j}{\omega_i} e^{-\Delta E/kT} q_{ji} \quad (20.16)$$

Also, if either  $S = 0$  or  $L = 0$ , then there is a simple relation for the collision strength between a term with a singlet level and a term containing several levels, *i.e.*,

$$\Omega(SLJ, S' L' J') = \frac{(2J' + 1)}{(2S' + 1)(2L' + 1)} \Omega(SL, S' L') \quad (20.17)$$

For example, let's look at the energy levels of doubly ionized oxygen, The ground state of  $O^{++}$  is a “p-squared” configuration, *i.e.*, the electrons are in  $1s^2 2s^2 2p^2$ . In the  $2p^2$  shell are 5 different energy levels: the ground state term is at  $L = 1$ , and, depending on the spins of the electrons, the term has three different energy states,  ${}^3P_0$ ,  ${}^3P_1$ , and  ${}^3P_2$ . A couple of eV above these  $P$  states is a  ${}^1D_2$  state, and about 3 eV above that is a singlet  $L = 0$  state,  ${}^1S_0$ . All these states have  $n = 2$ , so transitions between them are classically forbidden; in practice, their  $A$  values are  $\lesssim 0.1 \text{ sec}^{-1}$ .

Because the  $D$  state and the  $S$  state are both singlets, a single number can be given for the collision strength between  ${}^3P$  and  ${}^1D$ : in this case,  $\Omega({}^3P, {}^1D) = 2.17$ . The collision strength from the individual levels are therefore

$$\begin{aligned}\Omega({}^3P_2, {}^1D) &= \frac{5}{9}\Omega({}^3P, {}^1D) \\ \Omega({}^3P_1, {}^1D) &= \frac{3}{9}\Omega({}^3P, {}^1D) \\ \Omega({}^3P_0, {}^1D) &= \frac{1}{9}\Omega({}^3P, {}^1D)\end{aligned}\tag{20.18}$$

If the density of particles is low, then it is unlikely that a collisional de-excitation will occur before a spontaneous decay occurs. In this case, every collisional excitation creates a photon that escapes the nebula. The total collisional cooling for a given species is then

$$L_C = \sum_j N_e N_i q_{1,j} h\nu_{1,j}\tag{20.19}$$

where the sum is taken over all excited levels (typically 4), and  $h\nu_{1,j}$  is the energy difference between the ion's ground state and state  $j$ .