

Photometric Mass-to-Light Ratio

In addition to a population's total luminosity, there are several other quantities which can be computed from first principles. First is the photometric mass-to-light ratio. Recall the total luminosity of main sequence stars in a stellar population is

$$\mathcal{L}_D = \frac{M_0 \phi_1 \ell_1}{\alpha - x} \left\{ \left(\frac{m_{tn}}{m_1} \right)^{\alpha - x} - \left(\frac{m_L}{m_1} \right)^{\alpha - x} \right\} \quad (15.06)$$

where m_{tn} is the turnoff mass of the main sequence. As stated previously, $\alpha - x$ is usually greater than zero, so that last term of this equation is negligible; stars at the low end of the mass sequence do not contribute significantly to the cluster's luminosity.

On the other hand, low mass stars can be an important part of the cluster's mass. Consider the definition of the IMF in (15.02)

$$\begin{aligned} \mathcal{M} &= M_0 \int_{m_L}^{m_U} \phi(m/m_1) m d(m/m_1) \\ &= M_0 \int_{m_L}^{m_U} \phi_1 (m/m_1)^{-x} d(m/m_1) \\ &= \frac{M_0 \phi_1}{1 - x} \left\{ \left(\frac{m_U}{m_1} \right)^{1-x} - \left(\frac{m_L}{m_1} \right)^{1-x} \right\} \end{aligned} \quad (16.01)$$

where m_U is the upper mass cutoff for star formation (somewhere between 60 and $100 \mathcal{M}_\odot$). If $x > 1$ (as is normally assumed), the exponents in (16.01) are less than zero, and the value of \mathcal{M} is dominated by the last term in the equation. Physically, this means that most of the stellar mass of population resides in low mass stars.

The combination of (15.06) and (16.01) means that it is virtually impossible to determine a population's photometric mass-to-light. One can always drive up \mathcal{M}/\mathcal{L} by invoking the existence of low-mass stars which add mass (via 16.01), without luminosity (15.06).

Stellar Evolutionary Flux

An interesting property of stellar populations is the stellar evolutionary flux, *i.e.*, the number of stars passing through any (post main-sequence) phase of evolution at a given time. This is controlled by the rate at which stars turn off the main sequence. (All other rates are much faster than this.) As we have seen previously, the rate of stars turning off the main sequence is

$$N_{tn} = M_0 \phi(m_{tn}/m_1) \frac{dm_{tn}}{dt} = M_0 \phi_1 \left(\frac{m}{m_1} \right)^{-(1+x)} \frac{d(m_{tn}/m_1)}{dt} \quad (16.02)$$

From (15.05)

$$\frac{m}{m_1} = \left(\frac{t}{\tau_1} \right)^{1/1-\alpha} \implies \frac{d(m_{tn}/m_1)}{dt} = \frac{1}{\tau_1(1-\alpha)} \left(\frac{t}{\tau_1} \right)^{\alpha/(1-\alpha)} \quad (16.03)$$

so the stellar evolutionary flux is

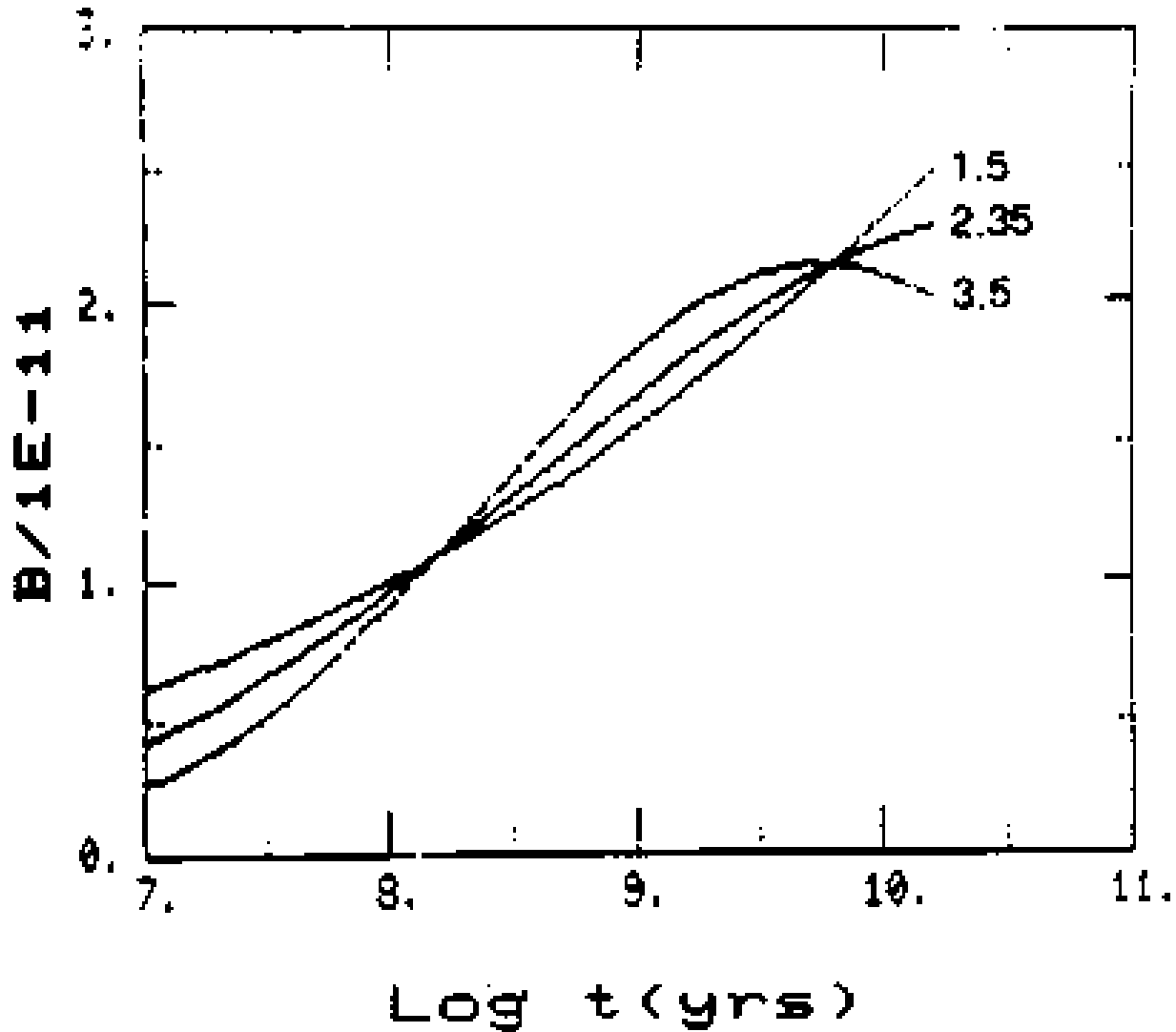
$$N_{tn} = \frac{M_0 \phi_1}{\tau_1(1-\alpha)} \left(\frac{t}{\tau_1} \right)^{\frac{\alpha-x-1}{1-\alpha}} \quad (16.04)$$

Naturally, the number of stars evolving through any phase of evolution in a galaxy is proportional to the number of stars in the galaxy. The best way to take this dependence out is to normalize the evolutionary flux to the size of the galaxy. This cannot be done with mass, since we do not know the total number of stars that are present in the system. However, we do know the population's luminosity, which is defined in (15.16). So, when we normalize (16.04) to galaxy luminosity, we arrive at the population's **luminosity-specific stellar evolutionary flux**. After a bit of math, this quantity comes out to be

$$b = \frac{N_{tn}}{L_t} = \frac{\alpha - x}{(\alpha - 1)(1 + G(t))\ell_1\tau_1} \left(\frac{t}{\tau_1} \right)^{\frac{1}{\alpha-1}} \quad (16.05)$$

Note that the exponent of time is less than 1; thus b is rather insensitive to the age of the stellar population. (The difference between the stellar evolutionary flux of a 7 Gyr stellar population and a 12 Gyr population is less than a 25%.) Similarly, the stellar evolutionary flux is not very sensitive to the exact value of x .

Detailed calculations show that the luminosity-specific stellar evolutionary flux is $b \approx 2 \times 10^{-11}$ stars $\text{yr}^{-1} L_{\odot}^{-1}$ for all old (or moderately old) stellar populations. This makes it easy to predict the number of any post-main sequence type of star present in a galaxy: all you need to know is b , the luminosity of the galaxy, and the lifetime of the phase in question. For example, the lifetime of a planetary nebula is $\tau \sim 25,000$ years. If a galaxy is observed to have an absolute luminosity of $\mathcal{L} = 10^{11} \mathcal{L}_{\odot}$, then the number of PN in the galaxy is $N(PN) \approx b \cdot \mathcal{L} \cdot \tau = 50,000$.



The plot above shows how a population's stellar evolutionary flux (stars evolving through a particular phase of evolution per unit time per unit bolometric luminosity of the population) changes with the system's age and IMF. Note that the x-axis (population age) is a log quantity, while the y-axis is linear. An old ($\gtrsim 3$ Gyr) stellar population with an absolute (bolometric) luminosity of $10^{11} \mathcal{L}_{\odot}$ has about 2 stars per year turning off the main sequence, independent of the system's age or initial mass function.

Mass Loss from Stars

Another useful quantity to know is the rate of mass loss from stars as a function of time. As before, the key to calculating this is to realize that almost all the mass lost from stars comes during the post main-sequence phase of evolution. Thus, the rate at which a population of stars loses mass is just by the rate at which stars move off the main sequence times the amount of mass each star loses. If m is the initial mass of the star, and w the mass of the stellar remnant (*i.e.*, white dwarf), then the mass ejection rate is

$$E(t) = N_{tn}(m_{tn} - w) = M_0\phi(m_{tn})\frac{dm_{tn}}{dt}(m_{tn} - w) \quad (16.06)$$

After performing the same substitutions as was done for the calculation of stellar evolutionary flux, this simplifies to

$$E(t) = \frac{M_0\phi_1}{\tau_1(1 - \alpha)}(m_{tn} - w) \left(\frac{t}{\tau_1}\right)^{\frac{\alpha - x - 1}{1 - \alpha}} \quad (16.07)$$

Once again, if we normalize to luminosity via (15.16), we can derive the luminosity specific mass loss rate

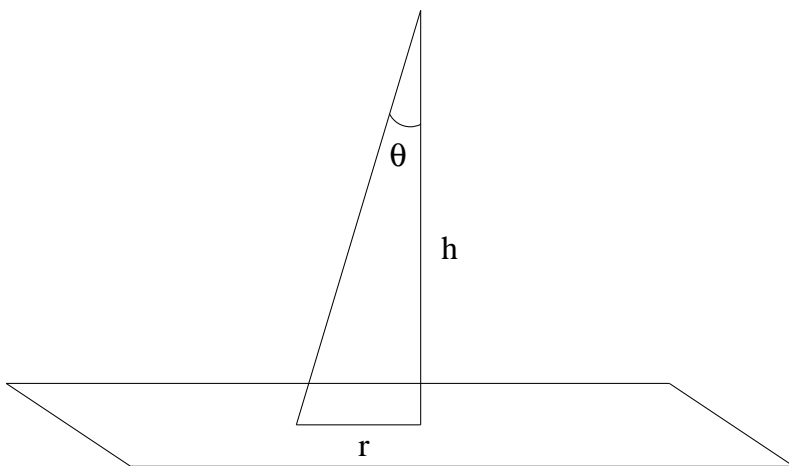
$$\frac{E(t)}{\mathcal{L}_t} = \frac{\alpha - x}{1 + G(t)} \frac{(m_{tn} - w)}{\alpha - 1} \frac{1}{\ell_{tn}t} \quad (16.08)$$

Numerically, this works out to a mass loss rate of $\sim 0.02\mathcal{M}_\odot$ per Gigayear per unit solar luminosity for an old stellar population that is 10^{10} years old. When integrated over the lifetime of a galaxy, the result is that $\sim 15\%$ of the original mass will be lost from stars during a Hubble time.

Galaxy Sweeping

As we have seen, the mass lost from stars in elliptical galaxies thermalizes at x-ray temperatures. If the gas is then heated further via energy input from supernovae, and if the mass of the galaxy is small enough ($\mathcal{M} \lesssim 10^{11} \mathcal{M}_\odot$), then the x-ray gas' temperature will be greater than that needed to escape the gravitational pull of the galaxy. The galaxy will develop a “Matthews-Baker” wind, and the gas will find its way into the potential of the cluster.

Now consider a spiral galaxy moving through a cluster. For simplicity, let's approximate the spiral galaxy as an infinite plane, with surface mass density of σ_* . From freshman physics, the force required to lift of particle of mass m from the surface of the galaxy to infinity is



$$\begin{aligned} \frac{F}{m} &= \int_0^\infty G \cdot \frac{2\pi r \sigma_*}{(r^2 + h^2)} \cos \theta dr = \int_0^\infty \frac{2\pi r \sigma_* G}{(r^2 + h^2)} \cdot \frac{h}{(r^2 + h^2)^{1/2}} dr \\ &= \int_0^\infty 2\pi G \sigma_* h \cdot \frac{r}{(r^2 + h^2)^{3/2}} dr = 2\pi G \sigma_* \end{aligned} \quad (16.09)$$

Now let the surface density of gas in the plane of the spiral galaxy be σ_{gas} . Since the mass of gas contained within a surface area dA is related to the surface density by $dm = \sigma_{\text{gas}} dA$, the pressure required to strip gas from the galaxy is

$$P = \frac{F}{dA} = 2\pi G \sigma_* \sigma_{\text{gas}} \quad (16.10)$$

Now when the galaxy moves through the cluster, the gas in the galaxy will feel a ram pressure due to x-ray gas. If the density of the x-ray gas is ρ_x , then, from fluid mechanics, the ram pressure is

$$P = \rho_x v^2 \quad (16.11)$$

Consequently, the gas inside the galaxy will get stripped out by the ram pressure when

$$\rho_x v^2 > 2\pi G \sigma_* \sigma_{\text{gas}} \quad (16.12)$$

This is the Gunn-Gott criterion for sweeping of gas from spirals. When does this occur? Well, there are $\sim 2 \times 10^{11}$ stars in the Milky Way, and the size of the Milky Way's disk is $R \sim 20$ kpc. If a typical Milky Way star is one solar mass, then the approximate stellar surface density is $\sigma_* = 2 \times 10^{11} / \pi R^2 \sim 0.03$ gm cm $^{-2}$. Also, if the density of particles in the local interstellar medium is 1 atom cm $^{-3}$ and the thickness of the disk is 200 pc, then the surface density of gas is about $\sigma_{\text{gas}} \sim 6 \times 10^{20}$ atoms cm $^{-2}$. Plugging these numbers into (16.12), and picking a typical velocity for a galaxy moving through a cluster ($v \sim 1300$ km s $^{-1}$), yields a requisite x-ray gas density of 5×10^{-4} atoms cm $^{-2}$.

The above shows that it does not take much of an intercluster medium to cause spirals to be stripped of their gas disk. When this occurs, star formation in the spirals will rapidly come to a halt (since they'll be no gas to form stars), and the galaxy will (presumably) change into an S0 lenticular galaxy. At the same time, the increased amount of intracluster material will cause the next spiral galaxy to be stripped that much easier. As time goes on, therefore, the amount (and x-ray luminosity) of the intracluster medium will be increased substantially.