Astronomy 485 — Problem Set 5 — Weeks 10 and 11

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Problem 3 is worth 20 points. All other problems are worth 10 points.

1. (a) A pulsar is observed to have a gamma-ray luminosity of L watts and a pulse period of P seconds. Estimate a lower limit on $\dot{P}$. If both P and $\dot{P}$ are measured discuss how a limit on the distance to the pulsar can be obtained from its gamma-ray flux (i.e., calculate an inequality that the distance must satisfy). You may assume isotropic gamma-ray emission, although this may not be the case in reality.

1. (b) Estimate the scale height (the distance over which the density increases by a factor of e) of the (iron) atmosphere of a 1.4 $\mathrm{M}_\odot$ neutron star with surface temperature 10^6 K. You may assume the atmosphere is an ideal gas.

1. (c) Suppose a spherical body spinning originally at a rate ω_1 with a radius R_1 collapses gravitationally to a radius R_2, conserving its mass M and angular momentum J. Express the ratios of the new and old spin rates ω_2/ω_1 and the new and old rotational energies E_2/E_1 in terms of the ratio R_1/R_2 (you may assume the moment of inertia for a homogeneous sphere). By what factor would the core of a star spin faster if it were to collapse from a radius typical of a white dwarf to the dimensions typical of a neutron star? By what factor would the rotational energy increase in such a collapse? Where ultimately does this energy come from?

2. (a) A neutron star cannot spin with less than a certain critical period or it will start to shed mass from its equator due to centrifugal force. Consider a neutron star of mass M and radius R. Show that it will shed mass if its period is less than

$$P_{\mathrm{min}} = K \left(\frac{1.4\ M_\odot}{M} \right)^{1/2} \left(\frac{R}{10\ \mathrm{km}} \right)^{3/2} \mathrm{ms}$$

where K is a constant. You may assume Newtonian gravity, and you may neglect deformation of the neutron star due to its rotation and magnetic field. What is the numerical value of K?

2. (b) A more detailed calculation that includes General Relativity and other effects gives $K = 0.77$ (see *Nature*, 340, 617 if you are interested). Look at the World Wide Web site `http://www.atnf.csiro.au/research/pulsar/psrcat/` to find the name and period of the pulsar with the shortest spin period. Using the equation above with $K = 0.77$ and $M = 1.4\ M_\odot$, calculate the limit on the radius of this neutron star. This limit is important because it constrains the equation of state for neutron star matter.

3. Based on the assigned reading for Week 10, please answer the following questions. Long calculations are *not* needed.

3. (a) How might the eccentric orbit of a massive X-ray binary arise?

3. (b) How did people determine the nature of the companion star in the 4U 1820–30 system?

3. (c) What is a dipper? Why are dips confined to just part of the binary orbit?

3. (d) What is the proposed theoretical explanation for the M15/AC211 system?

3. (e) What are some of the special ways that LMXBs can be produced in a globular cluster environment?

3. (f) How can we roughly constrain the radius of a neutron star using an X-ray burst? What element burns explosively during an X-ray burst?

3. (g) How and why are the stellar winds in 4U 1700–37 and Cen X-3 different? What is the apparent 'flaring' behavior of 4U 1700–37 thought to be due to?

3. (h) Define the quantity η. What is its numerical value for a white dwarf and a neutron star? Are there any physical processes that have $\eta = 1$? If so, please name them.

3. (i) What is the nickname for 4U 1624–49 and why is this nickname appropriate?

3. (j) What is the origin of photospheric radius expansion during an X-ray burst?

3. (k) Why is the extremely high X-ray luminosity of A 0538–66 somewhat surprising? What do its P-Cygni line profiles indicate? Why do its weak magnetic field and highly eccentric orbit seem to be in conflict?

3. (l) What is an accretion wake? Is it inside or outside the magnetospheric radius?

3. (m) Why might Vela X-1 be relevant to understanding the nuclear equation of state?

3. (n) Why is PSR 1957+20 appropriately named the 'Black Widow Pulsar'?

3. (o) Why do we see relatively few X-ray eclipsing LMXBs?

3. (p) Why were QPOs initially difficult to discover in X-ray data?

3. (q) What famous X-ray source shows clear relaxation oscillations? What is the interpretation of these oscillations?

3. (r) Where do the optical emission lines originate from a Be star? What is the probable reason for this behavior?

3. (s) Why don't X-ray pulsars burst?

3. (t) How are millisecond X-ray pulsars created from LMXB?

4. (a) The eclipsing binary X-ray source SMC X-1 lies in the Small Magellanic Cloud. It is an X-ray pulsar. We detect ≈ 50 X-rays per second from it in a detector which has an aperture of 400 cm^2. The X-rays have a typical energy of 5 keV. Estimate the X-ray luminosity of SMC X-1.

4. (b) The orbital period of SMC X-1 is 3.892 days, and the maximum pulse time delay is ± 53.46 seconds. Optical spectroscopy suggests its companion to have a mass of $\approx 17 M_\odot$ and gives a radial velocity amplitude of 19 km s^{-1}. Estimate the mass of the neutron star. Note that this neutron star seems to be radiating either anisotropically or at a super-Eddington rate.

5. (a) Consider an LMXB that shows X-ray bursts. Estimate the interval between bursts if the quiescent accretion luminosity is 1% of $L_{\rm Edd}$. Assume a neutron star mass of 1.4 $M_\odot$ and a burst α parameter (see page 21 of the Fabian notes) of 70. You may model the burst as a 5-second 'spike' at $L_{\rm Edd}$.
5. (b) Use the Stefan-Boltzmann law to estimate the maximum blackbody temperature (in keV) reached during a burst.
5. (c) Estimate the maximum distance (in kpc) at which an old (single) neutron star can be detected by *Chandra* if it is moving through the interstellar medium at 20 km $\rm s^{-1}$ and has a mass of 1.4 $\rm M_\odot$. Take the number density of the interstellar medium to be 1 $\rm cm^{-3}$. The *Chandra* sensitivity level is about 4×10^{-16} erg $\rm cm^{-2}$ $\rm s^{-1}$ in a 10^5 s exposure. You may assume that the neutron star emits all of its flux in the *Chandra* band.

6. Please write an ≈ 2 page report on magnetars. Describe some recent discoveries about magnetars as well as the earlier work on these objects. I have put a relevant *Scientific American* article on the course World Wide Web page, and I'd suggest looking at `http://solomon.as.utexas.edu/~duncan/`, `http://arxiv.org/abs/astro-ph/0304205`, and `http://arxiv.org/abs/astro-ph/0502329`. Many other World Wide Web sites can be found using one of the standard search engines. Your report should address the following issues as well as other ones you think are important.

- What is the basic theoretical picture for magnetars? How might their extreme magnetic fields be created?
- What is the typical magnetic field of a magnetar, and how does this compare to the strongest magnetic fields that people can make?
- Why do magnetars spin so slowly?
- What important discovery was made on March 5, 1979, and how is it related to magnetars?
- What satellites and other facilities contributed to the discovery of the connection between soft gamma-ray repeaters and magnetars? How many soft gamma-ray repeaters have now shown pulsations?
- How do magnetars make soft gamma-ray outbursts? Do they follow the Gutenberg-Richter Law and, if so, why is this interesting?
- What happened on August 27, 1998 and on December 27, 2004?
- How many magnetars are currently known? How many more might there be in our Galaxy?

Please document your work by giving the references that you used.

7. The precise equation for gravitational wave radiation from a circular binary system is

$$L_{\rm GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}$$

where μ is the reduced mass and M is the total mass of the binary system (compare with page 26 of the Fabian notes).

Consider two dead pulsars (each of mass 1.4 $M_\odot$) in a circular orbit with separation a. Over time, the binary orbit will shrink due to gravitational wave radiation, and eventually the two dead pulsars will crash together (perhaps to make a classical gamma-ray burst—a totally different phenomenon from the soft gamma-ray repeaters you read about for Problem 6). Show that the time t_0 until $a \to 0$ is

$$t_0 = \frac{5}{256}\frac{c^5}{G^3}\frac{a_{\rm now}^4}{M^2\mu}.$$

For the orbital dynamics you may assume that Newtonian gravity is valid until $a \to 0$ (although in reality the situation is somewhat more complicated). Then show that, to order of magnitude, t_0 is given by

$$t_0 \approx 10^5 \left(\frac{P_{\rm now}}{1\ {\rm second}}\right)^{8/3} \ {\rm seconds}$$

where $P_{\rm now}$ is the observed orbital period. How long does a ten-minute binary system consisting of two dead pulsars last? If you are stuck you may want to look at pages 476–477 of *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects* by S.L. Shapiro and S.A. Teukolsky.

8. Based on the assigned reading for Week 11, please answer the following questions. Long calculations are *not* needed.

8. (a) How is the mass function related to the minimum mass of the X-ray emitting object in a binary system?

8. (b) Why is the average apparent speed of the jets from SS433 not equal to zero?

8. (c) Using Kepler's laws, show that a binary orbit interpretation for the optical line shifts of SS433 is implausible. The line shifts have a velocity amplitude of about 42000 km $\rm s^{-1}$ and a period of about 160 days.

8. (d) What is a possible (although unlikely) way to escape the argument that Cygnus X-1 contains a black hole?

8. (e) Why are ellipsoidal light curve variations useful for studying some black hole binaries? What is the basic idea behind studying these variations?

8. (f) What are some examples of emission-line stars? How do we efficiently find such stars?

8. (g) How have people accurately determined the distance to SS433?

8. (h) How does the kinetic luminosity of the SS433 jet compare to the observed photon luminosity of the system?

8. (i) Why is it thought that *EXOSAT* saw only one X-ray iron line from SS433?

8. (j) By up to approximately what factor does a black hole soft X-ray transient brighten (optically) during a typical outburst?