Astronomy 485 — Problem Set 4 — Weeks 7 and 8

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All problems are worth 10 points.

1. (a) Estimate the blast-wave velocity of a supernova remnant that is 3000 yr old and originally had 10^{44} J of kinetic energy. Estimate the temperature behind the blast-wave. The number density of the surrounding interstellar medium is 2×10^6 m^{-3}.
1. (b) Consider a circularized binary system where the stars have masses M_1 and M_2. Star 1 goes supernova and loses mass ΔM in the explosion. Show that ΔM needs to be at least $(M_1 + M_2)/n$ for the system to become unbound. Here n is an integer. What is the numerical value of n? You may assume that the mass loss occurs very rapidly compared to the orbital period, and you may neglect any effects from the collision of the supernova shell with star 2. Hint: Write down an equation for the total orbital energy, E, immediately after the supernova.
1. (c) *Chandra* has very high spatial resolution X-ray optics, allowing clear determination if an X-ray source is point-like or extended if its true angular diameter is 1 arcsecond or more. As such, it permits studies of X-ray emitting supernova remnants in external galaxies. Assume that a 10^{52} erg supernova (neglecting neutrinos) went off 300 years ago in the Andromeda galaxy (M31) in a region where the density of the interstellar medium is 1 cm^{-3}. What is the angular size of the remnant, and can *Chandra* determine if it is extended? What is the expected temperature of the ejecta?

2. (a) If 10^{46} J were released in neutrinos (of mean energy 10 MeV) from SN1987A in the Large Magellanic Cloud (LMC), how many passed through each person on Earth?
2. (b) If 10^{44} J were dumped into its 5 $M_\odot$ envelope what temperature or velocity could be obtained?
2. (c) Scientists have placed an upper limit on the mass of the (electron) neutrino based on the 19 neutrinos detected from SN1987A. They observed an ≈ 13 second spread of neutrino arrival times. If the neutrino has a mass, neutrinos of different energies will have different speeds. An instantaneous pulse of neutrinos with a variety of energies will spread out as it travels the large distance between the LMC and Earth, with the more energetic neutrinos arriving first. Show that the travel time of ultrarelativistic neutrinos of mass m_0 and energy E from a distance D to Earth is given approximately by

$$t \approx (D/c)[1 + (1/2)(m_0 c^2/E)^2].$$

Then show that the delay relative to a massless neutrino is

$$\Delta t \approx 2.5 \left(\frac{m_0 c^2}{10 \text{ eV}}\right)^2 \left(\frac{10 \text{ MeV}}{E}\right)^2 \text{seconds}$$

Use this equation with $\Delta t \approx 13$ seconds and $E \approx 13$ MeV to constrain the neutrino mass. More detailed considerations give a mass upper limit of ≈ 15 eV (see *The Astrophysical Journal*, 318, L63 for details if you are interested). The reason this is only an upper limit rather than a direct mass measurement is that it takes ~ 10 seconds for neutrinos to diffuse out of the hot, proto-neutron star. The next time a supernova goes off nearby, scientists hope to collect many more neutrinos so that they can study in detail the time structure of the neutrino emission and thereby look 'inside' a collapsing star. What an awesome sight awaits them!

3. Based on the assigned reading for Week 7 and Week 8, please answer the following questions. Long calculations are *not* needed.

3. (a) What is a plerion? What is the most famous example of a plerion? Is 0540–69.3 a plerion? Is the Cygnus Loop a plerion?

3. (b) What is the difference between a 'polar' and an 'intermediate polar'?

3. (c) In a supernova explosion, what does the outward moving shock do to iron nuclei in its path? What does this do to the shock?

3. (d) Briefly describe the formation of a cataclysmic variable.

3. (e) What is the evidence for accretion disks in dwarf novae?

3. (f) What are the peculiar features of Cas A (lovingly referred to by many X-ray astronomers as 'Crass A')?

3. (g) Why are many supernova remnants also bright infrared sources?

3. (h) What is the period gap? What is some of the speculation behind its origin?

3. (i) What is the approximate mass of the ejecta in Tycho's remnant? What are some of the main reasons this mass is only approximate?

3. (j) What are superhumps? How are they made?

4. Please read the article 'A Million-Second *Chandra* View of Cassiopeia A' by U. Hwang et al. (*The Astrophysical Journal*, 615, L117); this is available from the course World Wide Web page in Portable Document Format. Then please write an ≈ 2 page essay describing the main results of this article. You should address issues such as the following:

- Why was Cas A targeted for such an extensive *Chandra* observation? What properties does it have that made it a good observation target?
- What were some of the requirements and challenges associated with making these observations?
- What data-analysis techniques were utilized to reveal the bipolar structure of the ejecta in the clearest manner possible?
- What are the main physical implications deduced from the bipolar structure of the ejecta?
- What constraints do these data place upon the compact point source near the center of Cas A?
- Suppose you were a team member collaborating with Hwang and colleagues, and that you could choose a new project to pursue with these data (aside from the nature of the bipolar structure and the nature of the point source, which had already been 'claimed' by others). What project would you choose, and why? Briefly justify why your project would be of scientific interest.

5. **(a)** A white dwarf cannot spin with less than a certain critical period or it will start to shed mass from its equator due to centrifugal force. Estimate this critical period assuming a mean density of 10^8 g cm^{-3}.
5. **(b)** Making use of the de Broglie wavelength, please derive the approximate condition on n for (nonrelativistic) degeneracy in a perfect gas with particle mass m and temperature T. This derivation should only take about 3–5 lines, and do not worry about numerical factors of order unity.
5. **(c)** The electron number density in the Sun's core is 4.4×10^{31} electrons m^{-3}. Are the electrons degenerate? Are the (helium) nuclei associated with these electrons degenerate?

6. Degeneracy pressure becomes important if the mean separation of atoms is forced (e.g. by gravity) to be less than a_0. Estimate Δx in a degenerate star from the mass-radius relation (see page 7 of the Fabian notes), and by setting it equal to a_0, show that the mass of an object at the boundary between a degenerate star and a planet is $M_{\rm Ch}\alpha_{\rm f}^{3/2}$ ($\alpha_{\rm f}$ is the fine-structure constant and is $\approx \frac{1}{137}$). Do not worry if your answer is off by boring factors of order unity. Also elegantly write your answer in terms of the 'gravitational fine structure constant' $\alpha_{\rm G} = \frac{Gm_{\rm p}^2}{\hbar c}$. Compare this mass to the masses of the planets in the solar system. You may also find *Nature*, 278, 605 to be interesting (although you do not need it for this problem).

7. The most easily observed white dwarf in the sky is in the constellation of Eridanus. As you have read, three stars comprise the 40 Eridani system: 40 Eri A is a 4th-magnitude star similar to the Sun; 40 Eri B is a 10th-magnitude white dwarf; and 40 Eri C is an 11th-magnitude red M5 star. This problem deals with only the latter two stars, which are widely separated from 40 Eri A by 400 AU.
7. **(a)** The period of the 40 Eri B and C system is 247.9 years. The system's measured trigonometric parallax is $0.201''$ and the true angular extent of the semimajor axis is $6.89''$. The ratio of the distances of 40 Eri B and C from the center of mass is $a_{\rm B}/a_{\rm C} = 0.37$. Find the masses of 40 Eri B and C in terms of the mass of the Sun.
7. **(b)** The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in terms of the luminosity of the Sun. The absolute bolometric magnitude of the Sun is 4.76.
7. **(c)** The effective temperature of 40 Eri B is 16900 K. Calculate its radius, and compare your answer to the radius of the Sun and Earth. Calculate the average density of 40 Eri B.

8. Please look at the attached portion of the chart of the nuclides; this is also available from the course World Wide Web page.
8. **(a)** Starting from 'Br 79' at the lower left, use a red pen to show clearly the sequence of nuclides synthesized in a neutron-rich environment (neutron density of $n_{\rm n} = 10^7$ cm^{-3}) of a star with $T = 3 \times 10^8$ K. This is the typical 's-process' that we discussed in class. Note that your sequence can continue past unstable nuclei without β-decay if the half-life for β-decay is longer than $\sim$ 1000 years.
8. **(b)** Using a blue pen, please circle the nucleon numbers for all the (stable) 'r-process' nuclides that you would expect to be made as the result of a supernova explosion.
8. **(c)** Use a yellow highlighter to mark the remaining stable nuclides. Can you find a compelling astrophysical scenario for their origin (please give this an honest try but don't worry too much if you cannot)?