

Astronomy 485 — Problem Set 3 — Weeks 5 and 6

Niel Brandt

All problems are worth 10 points.

1. (a) Detector dead time. If the count rate from a bright X-ray source is high enough, the X-ray detector may miss a legitimate count because it is still processing a previous count. This is known as detector dead time; the detector is ‘dead’ (insensitive to further counts) for a time Δt after registering a count. Work out a simple equation for the actual count rate R' in terms of the measured count rate R and Δt . Counts that are missed do not themselves cause dead time. If we measure a source to have a count rate of $R = 100 \text{ count s}^{-1}$ and the dead time is $\Delta t = 1 \text{ ms}$, what is the actual count rate?

1. (b) The *ASCA* satellite weighed 420 kg. It was launched in 1993 by an ISAS M-3S-II rocket into a low Earth orbit with an altitude of about 550 km. Using Kepler’s laws, find the approximate orbital period that *ASCA* had (in minutes).

2. Ionized hydrogen accreted onto a white dwarf passes through a shock close to the surface at radius R_* . Show that the shock temperature is approximately given by

$$\frac{kT}{m_e c^2} = \frac{3}{32} \frac{m_p}{m_e} \frac{R_S}{R_*}.$$

Here $R_S = \frac{2GM}{c^2}$ is the Schwarzschild radius. As in class, you may treat the pre-shocked matter as if it were ‘cold.’ What is kT (in keV) if $R_* = 6000 \text{ km}$ and $M = 1M_\odot$?

3. Based on the assigned reading for Week 5, please clearly answer the following questions. Long calculations are *not* needed.

3. (a) When the *EXOSAT* observatory was at its apogee, how far was it from Earth compared to the mean distance the Moon is from Earth? During most of its orbit, was *EXOSAT* protected from high particle background by the magnetic field of the Earth?

3. (b) How did scientists definitively determine that the Crab Nebula is an X-ray source? What other important facts did they learn as a result of this identification observation?

3. (c) Name three of the first extragalactic X-ray sources found. Please use the NASA Extragalactic Database (NED; <http://nedwww.ipac.caltech.edu/>) to determine the B1950 coordinates (right ascensions and declinations) of these three sources. NED is a very useful tool that you might want to learn about.

3. (d) What is a quantum microcalorimeter?

3. (e) What is *Kvant*? What is *Granat* named after?

3. (f) What were the main scientific observing instruments on *ROSAT*?

3. (g) What is *Chandra* ACIS?

3. (h) Name the NASA ‘Great Observatories’ and the spectral bands in which they operate (or operated).

3. (i) Over what timescale does the ^{26}Al (1.8 MeV) line allow us to probe nucleosynthesis in the Galaxy? Why does it probe this timescale rather than a much shorter or much longer one?

3. (j) Are gamma-ray blazars thought to emit isotropically? Why or why not?

4. Based on the assigned reading for Week 6, please answer the following questions. Long calculations are *not* needed.

4. (a) What are some of the basic things X-ray spectroscopy has taught us about the lunar surface composition and how was this achieved?

4. (b) Given your knowledge about the planets, why might you expect Saturn to be a fainter *Chandra* source than Jupiter? There are at least two reasons. Neglect scattered solar X-rays.

4. (c) What is 40 Eri and why is it interesting from an X-ray point of view?

4. (d) Why do we think there is a large amount of starspot activity in RS CVn binaries? There are several reasons. Which of the two stars in the binary typically has the strongest starspot activity?

4. (e) What is AR Lac and how did people determine its coronal structure?

4. (f) What is the spectral type of most X-ray flare stars? What is the typical magnetic field in a spot on a flare star?

4. (g) The limiting 0.5–2 keV sensitivity of a moderate-length *Chandra* exposure is about 4×10^{-16} erg cm⁻² s⁻¹. For such an exposure, could *Chandra* detect a typical O star at the Galactic Center? In the Large Magellanic Cloud? In the Andromeda Galaxy? In the quasar PHL 1092 (use NED to get the redshift if needed)? You may neglect the effects of Galactic absorption, although in reality these can be quite important for such calculations.

4. (h) What is the Cygnus OB2 association and how did it surprise Rick Harnden? What is the great rift of Cygnus?

4. (i) What did η Carina do from 1830–1860? How much hot gas is needed to power the nebular emission from the Carina nebula?

4. (j) At the start of Section 5.4, Charles & Seward discuss two of the main phases of the interstellar medium: dilute, hot gas and cool, dense clouds. Show that these two phases are in approximate pressure equilibrium.

5. We have learned about the Eddington limit as the characteristic luminosity when gas is supported by radiation pressure against the gravitational attraction of a collapsed mass M . Here we will calculate a few other characteristic ‘Eddington quantities.’ Consider a black hole of radius $r_g = \frac{GM}{c^2}$. Consider Eddington limited accretion of ionized hydrogen onto this black hole with unit efficiency (for the conversion of mass into radiant energy). Hence we have $\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{c^2}$.

5. (a) The Eddington number density. Derive the characteristic number density n_{Edd} when matter is flowing radially through the black hole event horizon at nearly the speed of light (adopt a spherical accretion geometry). Show that

$$n_{\text{Edd}} \approx 1 \times 10^{19} \left(\frac{M}{M_{\odot}} \right)^{-1}$$

where n_{Edd} is measured in units of cm⁻³.

5. (b) The Eddington time. Calculate the numerical value (in years) of the quantity M/\dot{M}_{Edd} . What is the physical meaning of this quantity (i.e., by what factor does the black hole mass increase in this time)?

6. (a) The Cygnus superbubble is about 15° in diameter and is at a distance of ≈ 2 kpc. If its temperature is $\approx 2 \times 10^6$ K and its luminosity is 5×10^{36} erg s $^{-1}$, please estimate the energy content of the gas. You may assume that the superbubble is spherical and homogeneous. You may also assume solar abundances and neglect line emission. Could the energy content of the gas have come from just the photons emitted by one supernova?

6. (b) What is the cyclotron frequency of an electron in a starspot with a magnetic field of 4000 gauss? In what band of the spectrum is this frequency?

6. (c) The star Algol (β Per) is at a distance of 27 pc. An X-ray flare on Algol is observed to have a temperature of 6×10^7 K and a flux of 8×10^{-11} erg cm $^{-2}$ s $^{-1}$ at the Earth. If the radiative decay of the emission takes 25,000 s, what is the electron density, volume and radius of the emission region? You may assume a hemispherical geometry for the flare and a uniform density within this volume. You may also assume solar abundances and that bremsstrahlung dominates the flare emission.

7. Recall the fine structure constant (α) and the gravitational fine structure constant (α_G). We have seen how many important physical quantities contain these two constants.

7. (a) Show that the Rydberg constant can be written as $\frac{mc^2}{2}\alpha^2$. You should use cgs units for this problem.

7. (b) Show that the Bohr radius, the Compton wavelength, and the classical radius of the electron scale approximately as $1:\alpha:\alpha^2$. Do not worry about factors of 2π that may emerge as a result of h versus \hbar .

7. (c) In class we have used the Bohr radius several times. Here we will derive the Bohr radius in a simple way that will be useful later. Apply the uncertainty principle to a hydrogen atom where the electron and proton are separated by distance a . Show that its radius is $a_0 \approx \hbar^2/m_e e^2$ by minimizing the total energy (as a function of a). Do not worry if your answer is off by a numerical factor of less than ~ 10 . This problem does not require any highly technical quantum mechanics calculations.

Note: The point of this problem is to derive the Bohr radius approximately in a way different from that in most introductory physics books. Please do *not* follow the standard path presented in many introductory physics books but instead use the method above.

8. Please read the article ‘Confirmation of Earth-Mass Planets Orbiting the Millisecond Pulsar PSR B1257+12’ by A. Wolszczan (available from the course World Wide Web page in Portable Document Format). Although we have not yet covered pulsars in detail, this article should be clear, and it discusses some interesting data analysis methods. Then please write an ≈ 2 page essay describing the main results of this article. You should address issues such as the following:

- What are the basic observational and analysis techniques that were used to find the planets? How was chisquared fitting used in this work?
- How does this paper present very strong evidence that the planet interpretation is correct?
- How does the velocity resolution of pulsar timing compare to that of optical Doppler spectroscopy?
- How might these planets have formed?
- How might we find more such planets?